### Normal

The normal (or Gaussian) distribution is frequently used as a model of demand. It is characterized by two parameters, its mean $\mu$ and its variance $\sigma^2$.

The basic definitions and properties are

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \\ E[X] &= \mu \\ Var(X) &= \sigma^2 \\ \psi(s) &= e^{\mu s + \frac{\sigma^2 s}{2}}. \end{aligned}$$

The normal has the property that if $X$ and $Y$ are two independent normal random variables, then the sum $X + Y$ also has a normal distribution (it is "closed under addition"). For example, if $X$ and $Y$ are independent with means $\mu_X, \mu_Y$ and variances $\sigma_X^2, \sigma_Y^2$ (respectively), then their sum $X + Y$ has a normal distribution with mean $\mu_X + \mu_Y$ and variance $\sigma_X^2 + \sigma_Y^2$.

### Gumbel

The Gumbel (or double-exponential) distribution is frequently used in discrete-choice models because it is "closed under maximization." That is, the maximum of two Gumbel random variables is also a Gumbel random variable. It is characterized by two parameters, a scale parameter $\mu$ and location parameter $\eta$.

The basic definitions and properties are

$$\begin{aligned} f(x) &= \frac{1}{\mu} e^{-\frac{x-\eta}{\mu}} e^{-e^{-\frac{x-\eta}{\mu}}} \quad -\infty < x < \infty \\ E[X] &= \eta + \frac{1}{\gamma\mu} \\ Var(X) &= \frac{\mu^2\pi^2}{6} \\ \psi(s) &= e^{\eta s/\mu}\Gamma(1 + s\mu), \end{aligned}$$

where $\gamma \approx= 0.577$ is Euler's constant and $\Gamma(x)$ is the extension of the factorial function to real numbers

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt.$$

If $X_1$ and $X_2$ are two independent Gumbel random variables with parameters $(\eta_1, \mu)$ and $(\eta_2, \mu)$ respectively, then $\max\{X_1, X_2\}$ is a Gumbel random variable with parameters $(\mu(\ln(e^{\eta_1/\mu} + e^{\eta_2/\mu}), \mu)$.

## Stochastic Monotonicity and Convexity

Consider a random variable $X$ that depends on some parameter $\theta$, so that $X = X(\theta)$. That is, $X(\theta)$ is a *random function* of $\theta$. For example, $X$ could be the number